THERMO-MECHANICAL BEHAVIOR OF FOR STEEL BEAM CONCRETE SLAB COMPOSITE FLOORS

T. F. Yolaçan[1], S. Selamet[2]

[1]Department of Civil Engineering, Bogazici University, Istanbul, Turkey, firat.yolacan@boun.edu.tr
[2]Department of Civil Engineering, Bogazici University, Istanbul, Turkey, serdar.selamet@boun.edu.tr

Abstract

Thermomechanical behavior of steel framed concrete slabs mainly depends on in plane normal forces that are developed as a result of mean temperature increase and large deflections. In plane normal forces can be determined by solving two dimensional continuum and equilibrium equations. In this paper, simplified method has been developed to consider large deflection caused tensile membrane forces to the current closed form equations of thermo-mechanical behavior of plates. The finite element analysis were carried out to validate the developed equations. Results of these analysis validated that the developed equations are 98% accurate to determine beam mid-length deflections and 90% accurate to determine plate mid-point deflections in the case of large deflections. Considered the validated equations, simplified methodology is developed to determine thermomechanical behavior of steel framed concrete slab and it is tested for specific composite floor geometry subjected to ISO834 fire while carrying a $5kN/m^2$ live load.

Keywords: *Structural Fire, Composite Floor, Fire, Beam with Large Deflection, Plate with Large Deflection, Tensile Membrane Action*

1 Introduction

Steel-framed concrete slabs are commonly used in steel construction especially in high-rise buildings. An accurate estimation of fire performance of composite floor systems is therefore critical. While the finite element method has been widely used to determine the thermo-mechanical behavior of composite floors (Huang et al, 2000), developing these models is computationally expensive, time consuming and parametric studies are tedious. Due to these limitations, a simplified semi-analytical method is development in Matlab to estimate the thermo-mechanical behavior of composite floors.

In this study, an efficient and simplified algorithm is developed to calculate the deflection of the composite floor under the action of fire by taking account in plane normal forces due to mean temperature increase and large deflections due to thermal gradient. Once the deflection behavior is obtained, moments and tensile membrane forces can be estimated, hence the stresses developed along the section can be calculated. Knowing the axial and bending capacity of the composite floor, it becomes possible to properly design the floor subjected to fire.

The current closed form equation that is developed to determine thermo-mechanical behavior of the plates does not include large deflection caused in planes normal forces, also known as tensile membrane forces (Ugural, 1981). During fire, large vertical deflections develop due to thermal gradient within the composite floor cross section from the bottom flange of the steel beam to the top surface of the concrete slab (Rotter and Usmani, 2000).

2 Thermo-Mechanical Behavior of Concrete Slabs with Steel Beam in Composite Action

2.1 Beam Deflection with Membrane Force and Temperature Gradient Effect

In this section, the governing differential equation for the beam deflection under transverse loading, membrane (axial) force and temperature gradient is derived. Classical beam theory assumes that straight lines normal to the middle surface before deformation remain straight, normal to the middle surface and unchanged in length after the deformation. However, when the beam boundaries are restrained, in plane normal forces may arise as a result of temperature change or large deflections due to thermal gradient or large transverse loading and the assumption becomes invalid. Thus, Eq. 2.1.1 which represents governing differential equation for beam deflection under transverse loading must be revised to consider the axial forces.

$$EI\frac{\partial^4 w}{\partial x^4} = p \tag{2.1.1}$$

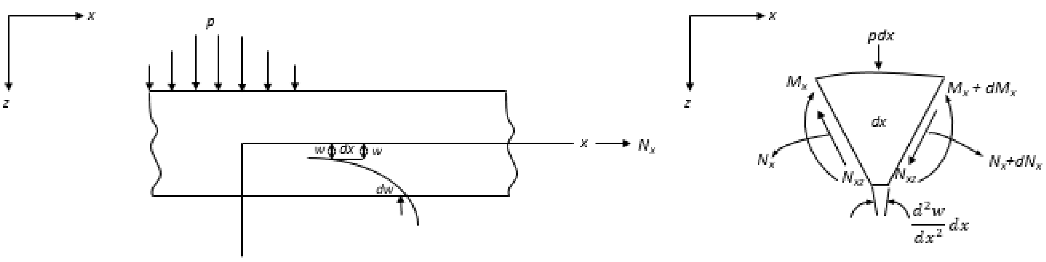

Figure 1. (a) Beam element subjected to transverse loading (b) Internal forces acting on infinitesimal beam length.

Figure 1 represents the beam element that is subjected to transverse loading and in plane direct force. The internal forces acting on the infinitesimal length of the beam element is shown in Figure 1b. Considering the equilibrium of the moments acting on infinitesimal length of the beam element and also knowing that as the limit d_x tends to zero and neglecting the shear force dN_{xz}, the equilibrium of the moments leads to Eq. 2.1.2 and Eq. 2.1.3.

$$N_{xz}dx - (M_x + dM_x) + M_x = 0 \tag{2.1.2}$$

$$N_{xz} = \frac{dM_x}{dx} \tag{2.1.3}$$

The condition $\sum F_z = 0$ then leads to Eq. 2.1.4.

$$dN_{xz} + pdx + N_x\frac{\partial^2 w}{\partial x^2}dx = 0 \tag{2.1.4}$$

Rearranging Eq. 2.1.4 by dividing dx and substituting the moment resultants that also includes thermally induced stress resultant as defined with Eq. 2.1.5 the governing differential equation for beam deflection subjected to the combined transverse loading, normal force and thermal gradient is obtained as Eq. 2.1.6.

$$M_x = EI\frac{\partial^2 w}{\partial x^2} - M^* \tag{2.1.5}$$

$$EI\frac{\partial^4 w}{\partial x^4} = p + \frac{\partial^2 M^*}{\partial x^2} + N_x\frac{\partial^2 w}{\partial x^2} \tag{2.1.6}$$

In which M^* is the thermal stress resultant

$$\mathrm{M}^* = E\alpha\int_{-t/2}^{t/2}\Delta T(z)zdz \tag{2.1.7}$$

2.2 Simply Supported Beam Subjected to Uniform Transverse Loading and Thermal Gradient

The governing beam deflection equation is solved for a simply supported beam in this section. It is possible to superimpose the deflection owing the temperature alone with those owing to transverse load. The boundary conditions of simply supported beam with length equals to L is shown with Eq. 2.2.1.

$$w = 0, u = 0, M_x = 0 \ at \ x = 0, L \tag{2.2.1}$$

For the boundary conditions shown with Eq. 2.2.1 the solution of the simply supported beam subjected to uniform transverse loading of p_o is found in Eq. 2.2.2.

$$w(x) = \frac{p_o x}{24EI}(L^3 - 2Lx^2 + x^3) \tag{2.2.2}$$

Due to superimposing of transverse loading and temperature varying over the thickness, the solution of Eq.2.1.6 reduces to Eq. 2.2.3.

$$EI\frac{\partial^4 w}{\partial x^4} = \frac{\partial^2 M^*}{\partial x^2} + N_x\frac{\partial^2 w}{\partial x^2} \tag{2.2.3}$$

Solution of Eq. 2.2.3 is obtained with representing thermal moment M^* and deflection w by the first term of a single Fourier sine series shown as Eq. 2.2.4 and Eq. 2.2.5 which of course satisfy the boundary conditions given with Eq. 2.2.1.

$$w(x) = w_T \sin\frac{\pi x}{L} \tag{2.2.4}$$

$$M^*(x) = M_1^* \sin\frac{\pi x}{L} \tag{2.2.5}$$

The Fourier coefficient w_T represents the mid-length deflection of a beam. In plane normal force $N_x$is sum of the thermal force N^* and the membrane force N_{xm} (Donnell, 1976), which are shown with Eq. 2.2.6 and Eq. 2.2.7.

$$N^* = E\alpha\int_{-t/2}^{t/2}\Delta T(z)dz \tag{2.2.6}$$

$$N_{xm} = EA\frac{\pi^2 w_T^2}{4L^2} \tag{2.2.7}$$

$$N_x = N^* + N_{xm} \tag{2.2.8}$$

The compression in-plane normal forces must be inserted with (-) sign and the tensile in-plane forces must be inserted with (+) in to the Eq. 2.2.8. Substituting Eqs.2.2.4, 2.2.5 and 2.2.8 in to Eq.2.2.3, the cubic equation for a mid-length of the simply supported beam subjected to temperature gradient is obtained (P. Khazaeinejad et al, 2015). Once the deflection of mid-length is calculated, deflection of any point on the beam length then can be calculated by Eq.2.2.4.

$$w_T^3 + \left(\frac{4A}{I} - \frac{4N^*L^2}{\pi^2 EA}\right)w_T + \frac{16M^*L^2}{\pi^3 EA} = 0 \tag{2.2.9}$$

2.3 Plate Deflection with Membrane Force and Temperature Gradient Effect

According to the fundamental assumption of the small deflection theory of bending for isotropic, homogeneous, elastic, thin plates based on the geometry of deformation the mid-plane does not stretch. However, when the mid-plane is strained subsequent to in plane normal forces, the fundamental assumption is no longer valid. If plate boundaries are restrained to lateral movement, the in plane normal forces may arise as a results of temperature change or large deflections due to thermal gradient or in the case of large transverse loading. Thus, Eq. 2.3.1 which represents governing differential equation for beam deflection under transverse loading should be revised to consider axial forces.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \tag{2.3.1}$$

Consider a plate element of sides dx and dy, under the action of direct forcesN_x, N_y and $N_{xy} = N_{yx}$which are functions of x and y only. Assume the body forces to be negligible. The top, front views of such an element and other resultants due to lateral force which also act on the element simultaneously are shown in Figure 2.

Considering the sum of in plane normal forces in the x direction shown in Figure 2, Eq. 2.3.2 is obtained.

$$(N_x + \frac{\partial N_x}{\partial x} dx) dy \cos \beta' - N_x dy \cos \beta \tag{2.3.2}$$

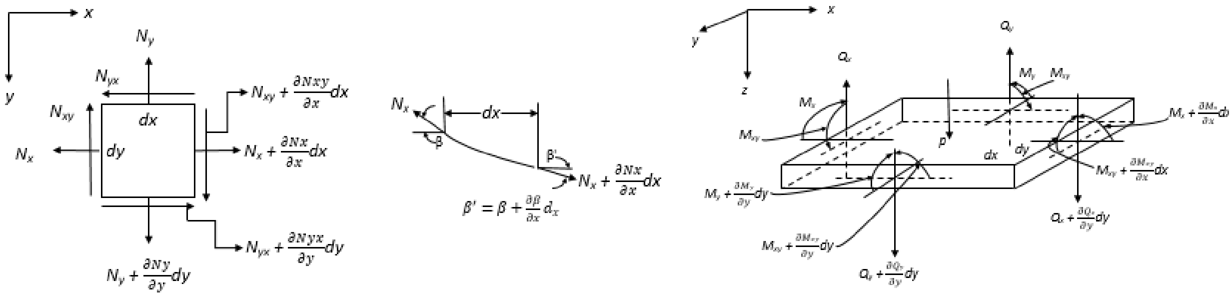

Figure 2. Stress resultants on plate element subjected to transverse loading and in plane normal forces.

For small β, Eq.2.3.2 reduces to $\left(\frac{\partial N_x}{\partial x}\right) dxdy$ and the sum of shear forces $N_{xy} dx$ is treated in similar way and the condition $\sum F_x = 0$ and $\sum F_y = 0$ then leads to Eqs.2.3.3a and Eq.2.3.3b.

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = 0 \tag{2.3.3a}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{2.3.3b}$$

To describe equilibrium in the z direction, it is necessary to consider the z components of the in-plane forces acting at each edge of the element. The z components of the forces acting on the x edges equals to Eq. 2.3.4 or with small deflection assumption Eq. 2.3.5

$$-N_x dy \sin \beta + \left(N_x + \frac{\partial N_x}{\partial x} dx\right) dy \sin B' \tag{2.3.4}$$

$$N_x \frac{\partial^2 w}{\partial x^2} dxdy + \frac{\partial N_x}{\partial x} \frac{\partial w}{\partial x} dxdy \tag{2.3.5}$$

The slope of the deflection in the y direction on the x edges equals to $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial y} + \left(\frac{\partial^2 w}{\partial x \partial y}\right) dx$. The z component of the shear forces N_{xy} and N_{yx} is obtained by Eq.2.3.6 and Eq.2.3.7.

$$N_{xy} \left(\frac{\partial^2 w}{\partial x \partial y}\right) dxdy + \frac{\partial N_{xy}}{\partial x} \frac{\partial w}{\partial x} dxdy \tag{2.3.6}$$

$$N_{yx} \left(\frac{\partial^2 w}{\partial x \partial y}\right) dxdy + \frac{\partial N_{yx}}{\partial y} \frac{\partial w}{\partial x} dxdy \tag{2.3.7}$$

For the forces in Figure 2, the condition $\sum F_z = 0$ then leads to Eq. 2.3.8.

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y}\right) \frac{\partial w}{\partial x} + \left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y}\right) \frac{\partial w}{\partial x} = 0 \tag{2.3.8}$$

It is observed that last 2 terms of the Eq.2.3.8 vanishes according to Eq.2.3.3a and Eq.2.3.3b. According to Figure 2, the equilibrium of the moments with respect to x and y axes is described by Eqs.2.3.9a and 2.3.9b.

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \tag{2.3.9a}$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \tag{2.3.9b}$$

Substituting moment resultants also including thermally induced stress resultants as defined Eq.2.3.10a, Eq.2.3.10b and Eq.2.3.10c (Ugural, 1981) to the Eq.2.3.9a and Eq.2.3.9b, the vertical shear forces Q_x and $Q_y$can be shown as Eq2.3.10d and Eq2.3.10e.

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \frac{v\partial^2 w}{\partial y^2}\right) - \frac{M^x}{(1-v)} \tag{2.3.10a}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial x^2} + \frac{v\partial^2 w}{\partial y^2}\right) - \frac{M^x}{(1-v)} \tag{2.3.10b}$$

$$M_{xy} = -(1-v)D\left(\frac{\partial^2 w}{\partial x \partial y}\right) \tag{2.3.10c}$$

$$Q_x = -D\left(\frac{\partial}{\partial x}\right)\nabla^2 \quad (2.3.10d) \qquad Q_y = -D\left(\frac{\partial}{\partial y}\right)\nabla^2 \quad (2.3.10e)$$

Inserting Eq.2.3.10d and Eq. Eq.2.3.10e into the Eq.2.3.8, the governing differential equation for deflection of thin plates subjected to the combined transverse loading, normal forces and thermal gradient is finally obtained.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(p + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} - \frac{1}{(1-v)}\nabla^2 M^*\right) \tag{2.3.11}$$

2.4 Simply Supported Plate Subjected to Uniform Transverse Loading and Thermal Gradient

As mentioned in Section 2.3, it is possible to superimpose the deflection due the temperature and due to transverse load. The boundary conditions of simply supported plate with edge dimensions a and b is shown below.

$$w = 0, u = 0, M_x = 0 \ at \ x = 0, a \tag{2.4.1a}$$

$$w = 0, v = 0, M_y = 0 \ at \ y = 0, b \tag{2.4.1b}$$

The solution of governing deflection equation of simply supported plate subjected to uniform transverse loading of p_o is found:

$$w = \frac{16p_o}{\pi^6 D(1-v)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m\pi x}{a}\sin\frac{n\pi x}{b}}{mn\left[\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 + \frac{N_x}{D}\left(\frac{m}{a\pi}\right)^2 + \frac{N_y}{D}\left(\frac{n}{b\pi}\right)^2\right]} \quad \text{for } m, n = 1, 3, 5 \cdots \tag{2.4.2}$$

The developed simplified method does not take account effect of shear forces $N_{xy} = N_{yx}$. Once the transverse loading p_0 and shear forces subtracted from Eq.2.3.10, it reduces to Eq. 2.4.3.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} - \frac{1}{(1-v)}\nabla^2 M^*\right) \tag{2.4.3}$$

Solution of Eq.2.4.3 is obtained with representing thermal moment M^* and deflection w by the Fourier sine series shown as Eq.2.4.4 and Eq.2.4.5 which of course satisfy the boundary conditions. Moreover, the condition shown with Eq.2.4.6 which is derived by Eqs.2.3.10a and 2.3.10b at boundaries must be satisfied.

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{2.4.4}$$

$$M^* = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} p_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{2.4.5}$$

$$D\nabla^2 \mathrm{w} = -\frac{M^x}{(1-v)} \tag{2.4.6}$$

Substituting Eqs.2.4.4, 2.4.5 in to Eq.2.4.3, the Fourier series equation to determine deflection at any point of simply supported plate subjected to temperature gradient over the thickness is obtained,

$$w = \frac{16M^*}{\pi^4 D(1-v)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m\pi x}{a}\sin\frac{n\pi x}{b}\left[(\frac{m}{a})^2+(\frac{n}{b})^2\right]}{mn\left[\left[(\frac{m}{a})^2+(\frac{n}{b})^2\right]^2+\frac{N_x}{D}(\frac{m}{a\pi})^2+\frac{N_y}{D}(\frac{n}{b\pi})^2\right]} \text{ for } m,n = 1,3,5 \cdots \quad (2.4.7)$$

where the normal forces are found as stated below. Eq. 2.4.8a and 2.4.8b will be explained in detail in Section 4.

$$N_x = \frac{\mathrm{N}^*}{(1-v)}\ \mathrm{N}^* + N_{xm} \quad (2.4.8a) \qquad N_y = \frac{\mathrm{N}^*}{(1-v)} + N_{ym} \quad (2.4.8b)$$

3. Validation of the Derived Governing Equations

To validate the solutions of equations developed under Section 2.2 and 2.4, a series of finite element analysis were carried out. The beam that is used to validate Eq.2.2.9 is comprised of $5m$ long, $300mm$ depth and $300mm$ width. Material properties of the beam is taken as C20 concrete material properties with constant thermal expansion coefficient and modulus of elasticity. The thermal gradient through the depth of the beam element is considered as $T_z = 1℃/mm$ where the mean temperature increases varies between $0℃$ to $300℃$ in increments of $50℃$.

The plate element that is used to validate Eq.2.4.7 is comprised of $5m$ long and $5m$ width and $200mm$ depth. Material properties of the plate is taken as C20 concrete material properties with constant thermal expansion coefficient and modulus of elasticity. The thermal gradient through the depth of the plate is taken as $T_z = 2℃/mm$ with mean temperature increases varies between $0℃$ to $150℃$ in increments of $25℃$.

Figure 3 shows that results of Eq.2.2.9 nearly same with finite element model results. Nearly 10% deviation obtained between the results of finite element model and Eq.2.4.7 for plate mid-point deflections in the case of large deflections. While using Eq.2.4.7, it is assumed that total membrane forces along the plate edges are uniformly distributed along the edges. Because the tensile membrane forces that are developed along the edges are related with square of the deflection, once the deflection of plate increases the deviation between the finite element model and Eq.2.4.7 also increases.

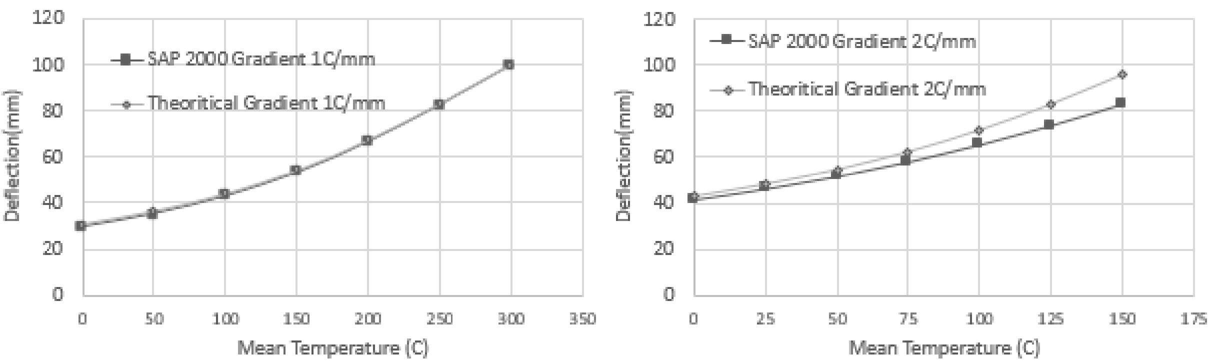

Figure 3.Mid-Point Deflection of Beam with $\mathrm{T_z} = 1℃/\mathrm{mm}$ and Plate with $\mathrm{T_z} = 2℃/\mathrm{mm}$

4. Simplified Method to Calculate Steel Framed Concrete Slab Deflection Behavior under Fire

The simplified method uses the equations that have been derived in Section 2.4.2 and 2.4.3 to calculate the deflection behavior of simply supported steel beam concrete slab composite floor. In the case of fire, the temperature distribution in the beam and the concrete slab differs across the composite section depth. Due to change in the mechanical properties in different temperatures, concrete slab is layered with equal length elements with respect to its depth and steel beam is layered as bottom flange, top flange and web. Each layer’s mechanical properties are calculated separately and transformed area method is used to find equivalent mechanical properties of the composite section.

When the fire initiates, temperature of steel beam increases very rapidly compared with the temperature of the concrete slab. As a result, the steel beam losses 80% of its stiffness when the beam average temperature reaches to $700℃$ and can therefore be neglected in the calculations. By knowing the temperature distribution along the depth of the composite section and calculating each layer mechanical properties with their respective temperature, the resultant force and moment in the composite cross section can be calculated using the transformed area method (Gere, 2004). Knowing the thermally induced in plane normal force and moment, Eq.2.2.9 and Eq.2.2.2 can be employed separately and results can be superimposed to find the mid-length deflection of the composite floor until the average temperature of steel beam reaches to $700℃$. Once the average temperature of steel beam reaches to $700℃$, the deflection behavior of the composite section is determined by superimposing results of Eq.2.4.2 and Eq.2.4.7 as mentioned before. However, to start to use Eq.2.4.2 and Eq.2.4.7 at time instant that average steel beam temperature is $700℃$, uniformly distributed in plane normal forces for the plate edges must be known.

Concrete slab can be thought as comprised of finite strips as shown with Figure 4 and mid-length deflections of these strips can be determined by Eq.2.4.7, and the tensile membrane forces in each strip is calculated by Eq.2.2.7. The tensile membrane forces in each strip are assumed to be uniformly distributed along the edge lengths. It is important to note that, the Eq.2.4.2 and Eq.2.4.7 will give the deflection of each point on the plate element without any initial curvature, hence the previously calculated deflection must be added to the deflection calculated by Eq.2.4.2.

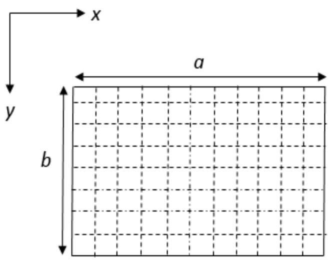

Figure 4. Finite Strips Demonstration of Rectangular Plate

The algorithm that describes the simplified method to determine the thermo-mechanical behavior of steel beam concrete slab composite floor subjected to fire is shown in Figure 5.

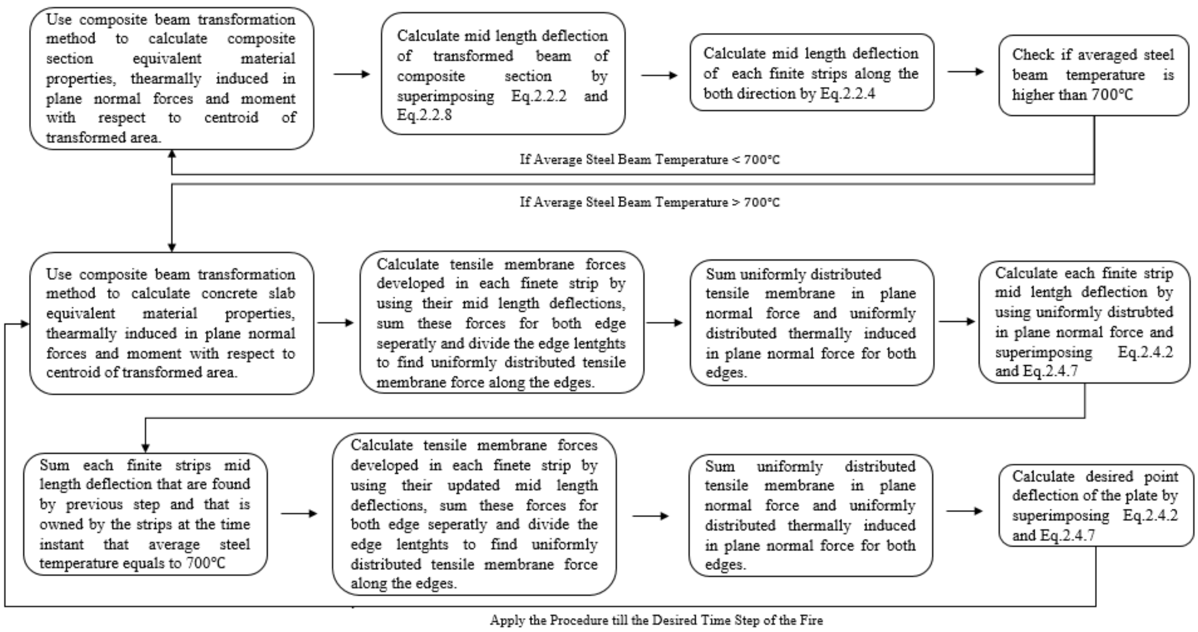

Figure 5. Simplified Method to Calculate Steel Framed Concrete Slab Deflection Behavior under the Action of Fire.

5. Results

Composite floor that is comprised of $5m$ length, $5m$ width, $200mm$ depth C20 concrete slab without reinforcement and $5m$ long IPE330 steel beam that is located at the mid length of the one of the edge is analyzed under the action of 120 minutes ISO834 fire. The temperature of the each layer of concrete slab and steel beam is obtained by Abaqus heat transfer analysis (DS-Simulia, 2010). Non-linear material behavior is taken into account by a reduced stiffness method with an efficient linear elastic calculation. Self-weight of the steel framed concrete slab is ignored and it is assumed that the composite floor carries $5kN/m^2$ live load.

Mid-point deflection behavior of the composite section is shown with Figure 6. The composite section starts to deflect with initiation of the fire and nearly at 8 minute of the analysis, the effect of the tensile membrane forces increases due to initiation of the large deflections and the rate of deflection reduces. When the average temperature of the steel beam reaches 700℃ at 21 minutes, sudden increase in the deflection is obtained due to assumption that steel beam losses its strength at 700℃. After that point, deflection behavior of composite floor is determined by the plate equation with considering uniformly distributed in plane tensile membrane forces and thermally induced in plane normal forces. Because large deflections have already developed in the concrete slab, deflection behavior is similar to the time instant just before the average temperature of the steel beam is 700℃. Due to the loss of the steel beam, no decrease in deflection is observed in the analysis. It is clearly seen from Figure 6 that the rate of deflection of the composite floor reduces in each time step because of the increasing tensile membrane force with large deflection.

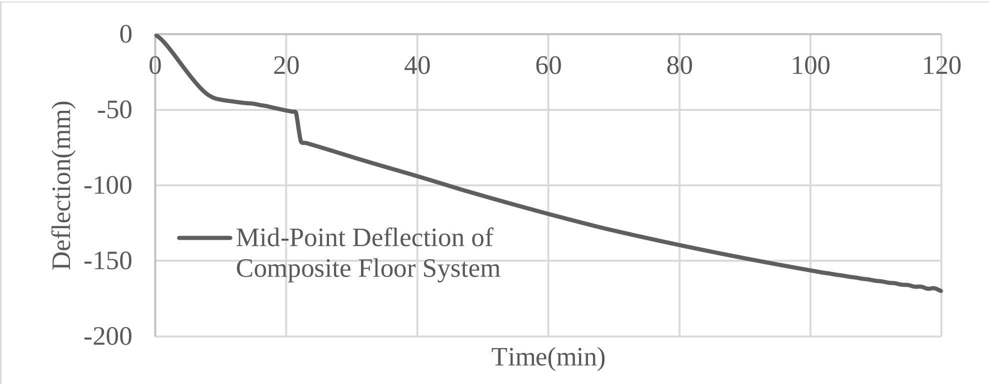

Figure 6. Mid-Point deflection of Composite Floor subjected to the ISO834 fire for 2 hours.

6. Conclusion

This paper presents an efficient method to determine deflection behavior of the steel beam-concrete slab composite floor subjected to fire. In this method, a computationally expensive finite element method is replaced by a fast and versatile semi-analytical method programmed in Matlab. It is assumed that plate is comprised of finite strips in both directions and the tensile membrane forces along these strips are calculated separately and distributed uniformly along the edges of the plates. At low mean temperature increase, the developed simplified method matches almost exactly with the finite element model solution, when the mean temperature increases the method is clearly capable of simulating the response of with reasonable accuracy.

The fast and reliable algorithm allows the structural fire engineers to study with different geometry and fire curves for the design of composite floors and therefore parametric studies become fast and reliable. Further modifications will be done in the future work on the methodology and the algorithm to consider effect of the steel deck, the reinforcement and material non-linearity with considering plastic strains.

7. References

Donnell, L.H. (1976), *Beams, Plates and Shells*, McGraw-Hill, New York.

DS-Simulia (2010), *Abaqus User's Manual*, Rhode Island.

Gere, J.M. (2004), *Mechanics of Materials,* Thomson Learning, Belmont.

Huang, Z., Burgess, I. W., Plank R. J. (2000) Three Dimensional Analysis of Composite Steel Framed Building in Fire, *Journal of Structural Engineers,* March 126(3), pp. 389-397.

Khazaeinejad P., Dai X., Usmani A. S.(2015), Analys,s of Heated Beams: Modelling Benchmark, *The First International Conference on Structural Safety under Fire & Blast*,September, pp: 469-473

Rotter, J.M, Usmani, A. S. (2000), Application of fundamental structural mechanics principle in assessing the Cardington restrained beam test, *Proceedings of First International Workshop in Structure in Fire*, September, pp: 14-30.

Ugural, A. C. (1981), *Stresses in, Plates and Shell,* McGraw-Hill, New York.